Chapitre 13. Les outils pour la géométrie

Corrigés des exercices À vous de jouer

Exercice d'application 1 page 259

Solution

a. On applique la formule du volume de la boule, $V = \frac{4\pi r^3}{3} = \frac{4\times 3^3}{3}\pi = 36\pi \approx 113{,}1$.

b. On applique la formule du volume du cylindre, $V = \pi r^2 h = 5^2 \times 4 \times \pi = 100\pi \approx 314{,}2$.

Exercice d'application 2 page 261

Solution

$\mathrm{AB}^2 = \mathrm{BC}^2 - \mathrm{AC}^2 = 13^2 - 8^2 = 105$

$\mathrm{AB} = \sqrt{105} \approx 10{,}25$

Exercice d'application 3 page 261

Solution

On a : $\frac{90}{\mathrm{AC}} = \frac{7}{12}$

D'où : $\mathrm{AC} = \frac{1080}{7}$

Exercice d'application 4 page 261

Solution

$\mathrm{AV} = \frac{4{,}5}{\cos(39°)} \approx 5{,}79$

Exercice d'application 5 page 262

Solution

$$\widehat{\mathrm{ACB}} = \sin^{-1}\left(\frac{7{,}8 \times \sin(51)}{6{,}9}\right)$$

$\widehat{\mathrm{ACB}} \approx 61{,}46°$

Exercice d'application 6 page 263

Solution

$JK = \sqrt{3^2 + 7^2 - 2 \times 3 \times 7 \times \cos(72)} \approx 6{,}71$

Exercice d'application 7 page 263

Solution

$p = 12 + 6{,}5 + 9{,}1 = 27{,}6$ donc $\frac{p}{2} = 13{,}8$

$\mathcal{A} = \sqrt{13{,}8 \times (13{,}8 - 12) \times (13{,}8 - 6{,}5) \times (13{,}8 - 9{,}1)} \approx 29{,}19$

Exercice d'application 8 page 265

Solution

On sait que $B(-2; -2)$.

$r_B = \sqrt{x_B^2 + y_B^2} = \sqrt{(-2)^2 + (-2)^2} = \sqrt{8} = 2\sqrt{2}$

$$\begin{cases} \cos(\theta_B) = \frac{x_B}{r_B} = \frac{-2}{2\sqrt{2}} = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2} \\ \sin(\theta_B) = \frac{y_B}{r_B} = \frac{-2}{2\sqrt{2}} = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2} \end{cases}$$

Pour identifier θ_B : $\arccos\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\pi}{4}$. Or $\frac{3\pi}{4}$ et $-\frac{3\pi}{4}$ ont pour cosinus $-\frac{\sqrt{2}}{2}$.

$\sin(\theta_B) < 0$ donc $\theta_B = -\frac{3\pi}{4}$.

Conclusion, on a $(r_B; \theta_B) = \left(2\sqrt{2}; -\frac{3\pi}{4}\right)$.

Exercice d'application 9 page 265

Solution

On sait que $D\left(4; \frac{\pi}{3}\right)$

$x_D = r_D \cos(\theta_D) = 4 \times \cos\left(\frac{\pi}{3}\right) = 4 \times \frac{1}{2} = 2$

$y_D = r_D \sin(\theta_D) = 4 \times \sin\left(\frac{\pi}{3}\right) = 4 \times \frac{\sqrt{3}}{2} = 2\sqrt{3}$

On a $(x_D; y_D) = (2; 2\sqrt{3})$.

Exercice d'application 10 page 267

Solution

$r_D = \sqrt{x_D^2 + y_D^2 + z_D^2} = \sqrt{1^2 + (0)^2 + 1^2} = \sqrt{2}$

$\sin(\varphi_D) = \frac{z_D}{r_D} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ or $\varphi_D \in [-\frac{\pi}{2}; \frac{\pi}{2}]$, on peut utiliser $\varphi_D = \arcsin\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$ $(radians)$.

$\begin{cases} \cos(\theta_D) = \frac{x_D}{r_D cos(\varphi_D)} = \frac{x_D}{\sqrt{x_D^2+y_D^2}} = \frac{1}{\sqrt{(1)^2+(0)^2}} = 1 \\ \sin(\theta_D) = \frac{y_D}{r_D cos(\varphi_D)} = \frac{y_D}{\sqrt{x_D^2+y_D^2}} = \frac{0}{\sqrt{(1)^2+(0)^2}} = 0 \end{cases}$ où $\theta_D \in [0; 2\pi[$.

Pour identifier θ_D : $\arccos(1) = 0$ $(radians)$ et on obtient $(r_D; \theta_D; \varphi_D) = \left(\sqrt{2}; 0; \frac{\pi}{4}\right)$.